A variational formulation for hyperspectral and multispectral image fusion

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1. Introduction

Due to technical constraints, satellite sensors cannot provide images with high spectral and spatial resolution at the same time but either a hyperspectral image (HS) or a multispectral one (MS). In this work we present a variational model for HS and MS image fusion. The novelty is the non-local regularization term that makes the fusion robust to additive noise. Moreover, we introduce a geometry constraint that forces the fused and the MS images to share high modulated frequencies.

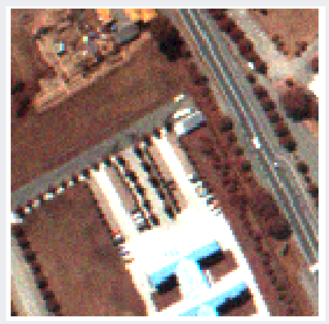
2. Data generation model

The data observation model

$$g_h = DBu_h + \varepsilon_h, \ \forall h \in \{1, \dots, H\},$$
$$f_m = (Su)_m + \varepsilon_m, \ \forall m \in \{1, \dots, M\},$$

- f is the MS image with M spectral bands and N pixels;
- g is the HS image with H spectral band ($H >> M$) and $N_l = \frac{N}{l^2}$ pixels where $l \in \mathbb{Z}^+$ is the sampling factor;
- $u = (u_1, \dots, u_H) \in \mathbb{R}^{H \times N}$ is the high-resolution HS image to reconstruct;
- ε_m and ε_h are realizations of Gaussian noises relative to f_m and g_h respectively;
- D and B are respectively a sub-sampling and a low pass filter that acts on each channel u_h, S is a spectral degradation operator.

Low-resolution hyperspectral (g)



High-resolution multispectral (f)

3. Proposed variational model

We propose the following nonlocal variational model for HS and MS fusion:

$$\min_{u \in \mathbb{R}^{H \times N}} \sum_{h=1}^{H} \|\nabla_{\omega_h} u_h\|_1 + \frac{\lambda}{2} \sum_{h=1}^{H} \|\tilde{P}_h u_h - P_h \tilde{g}_h\|_2^2 + \frac{\mu}{2} \sum_{h=1}^{H} \|DBu_h - g_h\|_2^2 + \frac{\gamma}{2} \sum_{m=1}^{M} \|(Su)_m - f_m\|_2^2,$$

- $\|\nabla_{\omega_h} u_h\|_1 = \sum_i |\nabla_{\omega_h} u_{h,i}|$ is a non-local regularization term where $|.|$ is the Euclidean norm and $(\nabla_{\omega_h} u_{h,i})_j = \sqrt{\omega_{h,i,j}}\,(u_{h,j} - u_{h,i})$;
- $\|DBu_h - g_h\|_2^2$ and $\|(Su)_m - f_m\|_2^2$ measure the deviation from the data observation model;
- $\|\tilde{P}_h u_h - P_h \tilde{g}_h\|_2^2$ is variational formulation of the radiometric constraint;
- λ, μ and γ are trade-off parameters.

4. Similarity weights

The non-local weight $\omega_{h,i,j}$ measures the similarity between two pixels i and j of channel h

$$\omega_{h,i,j} = \begin{cases} \frac{1}{\Gamma_i} \exp\left(-\frac{\|i-j\|_2^2}{h_{\text{spt}}^2} - \frac{\sum_{m=1}^{M} s_{m,h} \sum_{\{t \in \mathbb{Z}^2 : \|t\|_\infty \le \nu_c\}} \|f_{m,i+t} - f_{m,j+t}\|_2^2}{h_{\text{sim}}^2 (2\nu_c + 1)^2 \sum_{m=1}^{M} s_{m,h}} \right) & \text{if } \|i-j\|_\infty \le \nu_r \ (i \ne j) \\ 0 & \text{otherwise} \end{cases}$$

where h_{spt} and h_{sim} are filtering parameters, ν_r and ν_c determine the size of the search window and of the patch respectively and Γ_i is a normalization factor.

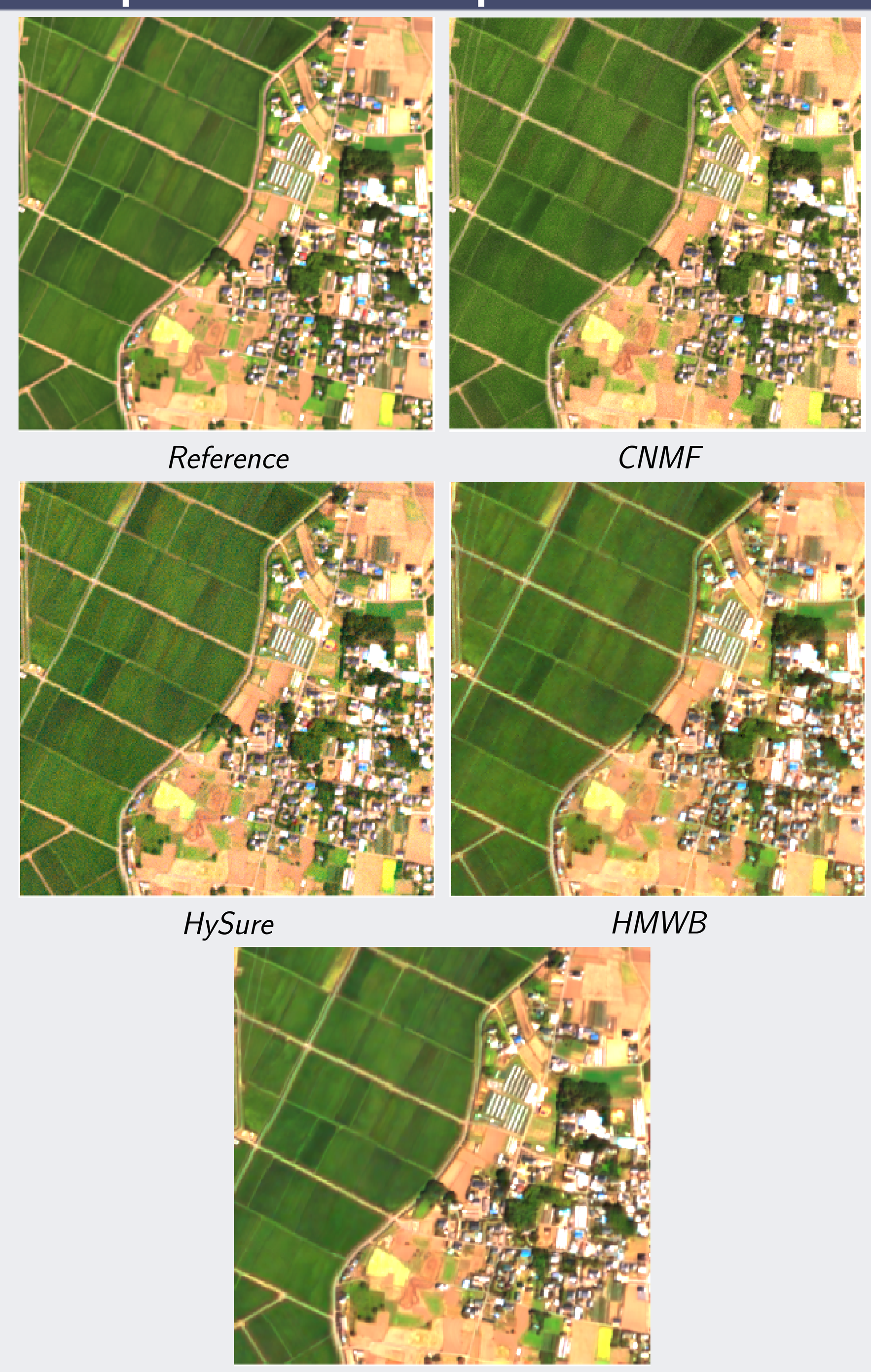
5. Radiometric constraint

The radiometric constraint forces the fused image to share the high spatial frequencies of the MS image [1] as follows:

$$\frac{u_h}{P_h} = \frac{\tilde{g}_h}{\tilde{P}_h} \Leftrightarrow u_h - \tilde{g}_h = \frac{\tilde{g}_h}{\tilde{P}_h}(P_h - \tilde{P}_h), \ \forall h \in \{1, \dots, H\}$$

- $P_h = \sum_{m=1}^{M} \frac{s_{m,h}}{s_h} f_m$ where $s_h = \sum_m s_{m,h}$;
- $\tilde{g}$ is the HS image upsampled to the high-resolution domain by bicubic interpolation;
- $\tilde{P}_h = \sum_{m=1}^{M} \frac{s_{m,h}}{s_h} \tilde{f}_m$ where $\tilde{f}$ is the MS image with the same spatial resolution as the HS image;

6. Experimental part and comparison



Chikusei data set (304 × 304 × 93). All results except ours are very affected by the noise and aliasing effects, e.g., see the color spots on the green trees located at the right border of the images.

	PSNR	ERGAS	SAM	$Q2^n$	CC	DD
Reference	∞	0	0	1	1	0
CNMF [2]	38.21	2.22	1.61	0.9576	0.9904	4.54
HySure [3]	41.69	**1.90**	1.49	0.9459	**0.9932**	2.99
HMWB [4]	40.42	2.05	1.54	0.9638	0.9921	3.08
Fused	**42.04**	1.94	**1.31**	**0.9697**	0.9931	**2.55**

Quantitative quality evaluation of each fused product on Chikusei dataset corrupted with low Gaussian noise. The DD values are provided in order of magnitude 10^{-9}.

- The spectral and the spatial resolutions of the fused image are visually close to the ones of the reference image;
- We can see that visually our fused image is less affected by noise than the other images of the state of the art;
- Our method outperforms the other state of the art methods in many quality indexes.
- The CC index is slightly lower that those provided by the other methods because the non-local regularization term is band decoupled.

7. Conclusions

In this work we have presented a new variational model for fusing hyperspectral and multispectral images. The proposed method compares favorably, visually and in terms of several quality metrics, to the state of the art and is more robust to noise. Future work will consist in introducing a channel-coupled regularization term in order to take into account and balance the inter-band correlation.

8. References

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