A New Mathematical Model for Pansharpening Satellite Images

Success Stories of Spanish Industrial Mathematics with Industry
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Introduction

- Satellite data $\begin{cases} \text{high-resolution } \textbf{panchromatic image} \text{ (PAN),} \\ \text{low-resolution } \textbf{multispectral image} \text{ (MS).} \end{cases}$

Introduction

- Satellite data {
  - high-resolution **panchromatic image** (PAN),
  - low-resolution **multispectral image** (MS).

Panchromatic image

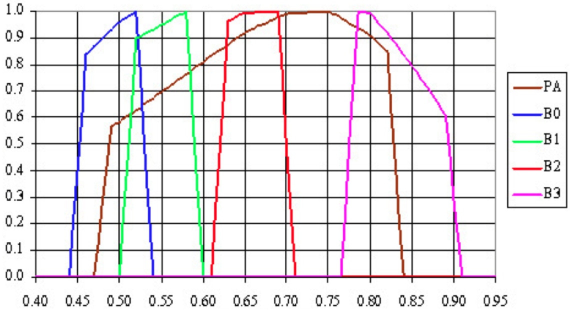
+

Low-resolution image

=

Pansharpened image

- Satellite data { high-resolution **panchromatic image** (PAN), low-resolution **multispectral image** (MS).



Spectral response of each sensor to wavelengths of light

Literature

- Academical framework:
  - Work with three channels (R, G, B).
  - PAN and MS spatially registered.
  - PAN obtained by linear combination of MS components.
- Far from real case but:
  - Good starting point.
  - It permits to simulate and compare with a ground truth.
- Real satellite data:
  - PAN sensor does not cover all MS wavelength ranges.
  - PAN and MS are misregistered.
  - MS suffers from aliasing and cannot be interpolated.

Literature

- **IHS transform:** convert RGB to IHS, replace I by PAN and get back to RGB.
- **PCA transform:** compute PC's, replace PC1 by PAN and reverse transform.
- **Brovey transform:** normalize MS image and multiply it by PAN.
- **Wavelet-based methods:** replace high-frequency coefficients by those of PAN.

- Convert RGB-image to IHS-image.

$$\begin{pmatrix} I \\ v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{\sqrt{2}}{6} & -\frac{\sqrt{2}}{6} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

- Replace I by PAN.
- Transformed back to RGB-space.

$$C = C_0 + (PAN - I), \quad C \in \{R, G, B\}$$

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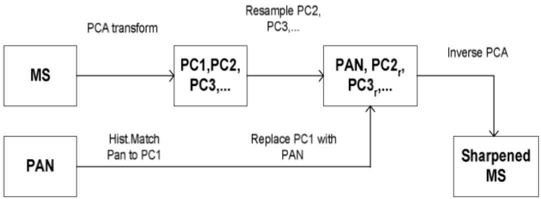

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$$C = \frac{C_0}{\frac{1}{3}\left(R_0 + G_0 + B_0\right)} PAN, \quad C \in \{R, G, B\}$$

Literature

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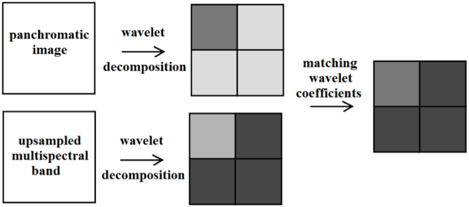


Variational framework

- **Variational methods:** Solve $\min_{u \in \mathcal{A}} J(u)$ s.t.

$$u \text{ with low energy } J \quad \Leftrightarrow \quad u \text{ satisfying desired properties}$$

- **Notations:**
  - $\Omega \subset \mathbb{R}^N$ open and bounded domain.
  - $S \subseteq \Omega$ sampling grid (low-resolution pixels).
  - PAN image: $P : \Omega \to \mathbb{R}$.
  - MS image: $\vec{u}^S = (u_1^S, \ldots, u_M^S)$, $u_m^S : S \to \mathbb{R}$, M spectral bands.
  - Pansharpened image: $\vec{u} = (u_1, \ldots, u_M)$, $u_m : \Omega \to \mathbb{R}$.

- Ballester *et al.*[1] proposed to minimize a functional with
  - a local regularization term transferring the geometry of panchromatic,
  - two fidelity terms with panchromatic and multispectral data.

[1] C. Ballester, V. Caselles, L. Igual, J. Verdera. *A variational model for P+XS image fusion*. Int. J. Comput. Vis., 69(1):43-58, 2006.

Variational framework

Assumptions

- PAN is a linear combination of multispectral channels.

$$P(x) \equiv \sum_{m=1}^{M} \alpha_m u_m(x), \quad \forall x \in \Omega,$$

where $\alpha_m \geq 0$ and $\sum_m \alpha_m = 1$.

- Low-resolution pixels formed from high-resolution ones by low-pass filtering followed by subsampling.

$$u_m^S(x) = k_m * u_m(x), \quad \forall x \in S, \quad \forall 1 \leq m \leq M.$$

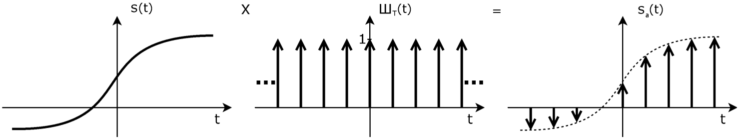
Variational framework

Ballester et al. proposed to minimize the **energy functional**

$$\begin{aligned} J(\vec{u}) &= \frac{1}{2}\sum_{m=1}^{M}\int_{\Omega}\left|\frac{\nabla P(x)^{\perp}}{\|\nabla P(x)\|}\cdot\nabla u_m(x)\right|^2 dx \\ &+ \frac{\lambda}{2}\int_{\Omega}\left(\sum_{m=1}^{M}\alpha_m u_m(x) - P(x)\right)^2 dx \\ &+ \frac{\mu}{2}\sum_{m=1}^{M}\int_{\Omega}\Pi_S\cdot\left(k_m * u_m(x) - u_m^{\Omega}(x)\right)^2 dx, \end{aligned}$$

where $\lambda, \mu > 0$ are trade-off parameters, and $\vec{u}^{\Omega} = (u_1^{\Omega}, \ldots, u_M^{\Omega})$ is an arbitrary continuous extension of $\vec{u}^S$ to the whole domain Ω.

$\Pi_S = \sum_{x\in S}\delta_x$ is a **Dirac's comb** defined by sampling grid S.



Proposed Variational Model I

We proposed to replace the local by a non-local regularization term

$$\begin{aligned} J(\vec{u}) &= \frac{1}{2}\sum_{m=1}^{M}\int_{\Omega}\int_{\Omega}\left(u_m(x)-u_m(y)\right)^2\omega(x,y)dxdy \\ &+ \frac{\lambda}{2}\int_{\Omega}\left(\sum_{m=1}^{M}\alpha_m u_m(x)-P(x)\right)^2 dx \\ &+ \frac{\mu}{2}\sum_{m=1}^{M}\int_{\Omega}\Pi_S\cdot\left(k_m * u_m(x)-u_m^{\Omega}(x)\right)^2 dx, \end{aligned}$$

where the **weights** are computed on PAN:

$$\omega(x,y)=\frac{1}{C(x)}e^{-\frac{d_\rho(P(x),P(y))}{h^2}}, \quad C(x)=\int_{\Omega}e^{-\frac{d(P(x),P(y))}{h^2}}dy,$$

s.t. $0\leq\omega(x,y)\leq 1$ and $\int_{\Omega}\omega(x,y)dy=1\ \forall x\in\Omega$.

Weights are non symmetric

Lemma

$J(\vec{u})$ is strictly convex for any $\vec{u} \in \mathcal{A}$.

Theorem

If $\vec{g} = (g_1, \ldots, g_M)$, with $g_m \in L^2(\Gamma)$, then $\exists!\ \vec{u}^* \in \mathcal{A}$ s.t.

$$J(\vec{u}^*) = \inf_{\vec{u} \in \mathcal{A}} J(\vec{u}).$$

For definitions of spaces $\mathcal{A}, \Gamma, \mathcal{Z}$ and respective norms see

A. Buades, B. Coll, J. Duran, C. Sbert, "A Nonlocal Variational Model for Pansharpening Image Fusion". SIAM J. Imaging Sci., vol. 7(2), pp:761-796, 2014.

- PAN given on $I = \{0, 1, \dots, N-1\} \times \{0, 1, \dots, N-1\}$.
- MS given on sampling grid $S \subseteq I$ of size $\frac{N}{s} \times \frac{N}{s}$, s sampling factor.
- **Discrete functional**:

$$
\begin{aligned}
J(\vec{u}) &= \frac{1}{2} \sum_{m=1}^{M} \sum_{p,q \in I} \left(u_m(p) - u_m(q)\right)^2 \omega(p,q) \\
&+ \frac{\lambda}{2} \sum_{p \in I} \left(\sum_{m=1}^{M} \alpha_m u_m(p) - P(p) \right)^2 \\
&+ \frac{\mu}{2} \sum_{m=1}^{M} \sum_{p \in S} \left(k_m * u_m(p) - u_m^{\Omega}(p)\right)^2.
\end{aligned}
$$

Explicit scheme for gradient descent method

$$\begin{aligned} u_m^{(n+1)}(p) &= u_m^{(n)}(p) - \Delta t \sum_{p,q \in I} \left(u_m^{(n)}(p) - u_m^{(n)}(q)\right)\left(\omega(p,q) + \omega(q,p)\right) \\ &- \Delta t \lambda \alpha_m \left(\sum_{k=1}^{M} \alpha_k u_m^{(n)}(p) - P(p) \right) \\ &- \Delta t \mu k_m^t * \left[\Pi_S \cdot \left(k_m * u_m^{(n)}(p) - u_m^{\Omega}(p) \right) \right], \quad \forall p \in I, \ \forall 1 \leq m \leq M. \end{aligned}$$

- $n \geq 0 \rightarrow$ iteration number.
- $\Delta t > 0 \rightarrow$ time step in the descent direction.
- $k_m \rightarrow$ Gaussian kernels of s.d. $\sigma = 1.2$ if $s = 2$ and $\sigma = 2.2$ if $s = 4$.
- Π_S is a $N \times N$ matrix s.t.

$$\Pi_S(p) = \begin{cases} 1 & \text{if } p \in S, \\ 0 & \text{if } p \notin S, \end{cases} \qquad \forall p \in I.$$

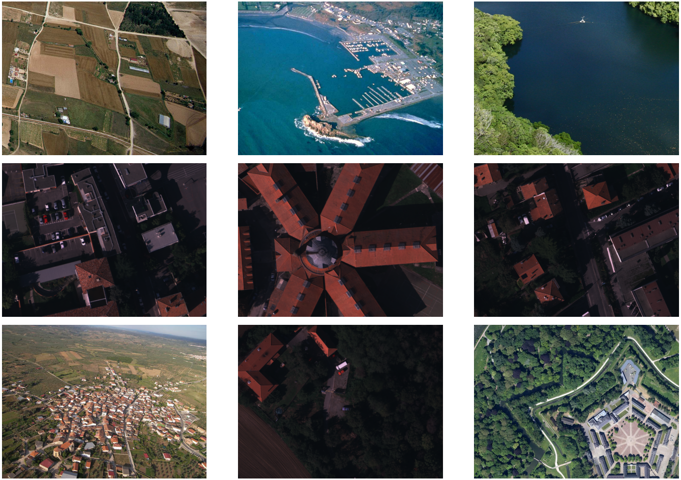
- **Discrete weights**:

$$\omega(p,q) = \begin{cases} \frac{1}{C(p)} e^{-\frac{1}{h^2}\sum_{t\in\mathcal{N}_0} \|P(p+t)-P(q+t)\|^2} & \text{if } \|p-q\| \le L, \\ 0 & \text{otherwise.} \end{cases}$$

- Regularity term restricted to pixels at a certain distance (7×7 pixels).
- $\mathcal{N}_0 \rightarrow l \times l$ window centered at $(0,0)$ (3×3 pixels).
- Weights-filtering parameter $\rightarrow h = 1.25$.
- Trade-off parameters $\rightarrow \lambda = 17.5$ and $\mu = 17.5 \cdot s^2$.
- Number of iterations $\rightarrow N_{iter} = 50$.

Proposed Variational Model I

Experimental Results on Simulated Data

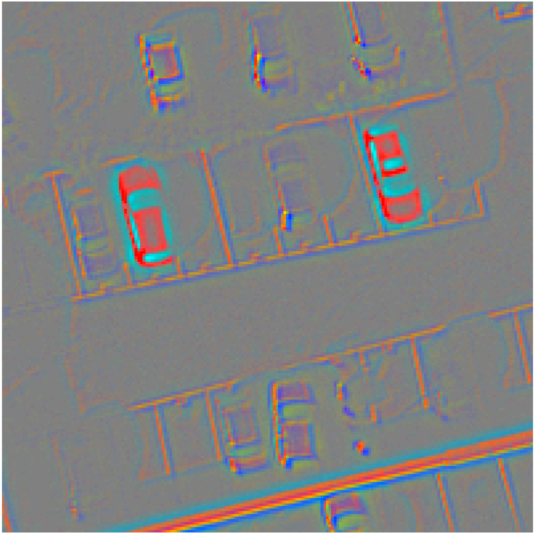


$s = 2$					$s = 4$				
IHS	BRV	WVL	PXS	NLV I	IHS	BRV	WVL	PXS	NLV I
1.16	1.14	1.80	1.54	1.05	1.93	1.91	2.26	2.15	1.60
1.42	1.40	2.58	1.97	1.37	2.23	2.21	2.96	2.64	1.83
1.03	0.99	1.90	1.43	0.87	1.72	1.68	2.17	1.94	1.34
2.08	2.07	2.71	1.92	1.68	2.62	2.62	2.89	2.54	2.26
2.43	2.43	3.21	2.24	2.10	3.02	3.02	3.44	2.86	2.52
1.42	1.41	1.96	1.26	1.18	1.83	1.82	2.11	1.70	1.49
1.75	1.74	2.92	2.46	1.45	3.04	3.03	3.68	3.47	2.29
0.99	0.98	1.46	0.88	0.82	1.33	1.32	1.59	1.22	1.04
1.27	1.25	2.36	1.95	1.14	2.21	2.20	2.86	2.60	1.63
1.51	1.49	2.32	1.74	1.30	2.21	2.20	2.66	2.35	1.78

Truth image

Proposed Variational Model I

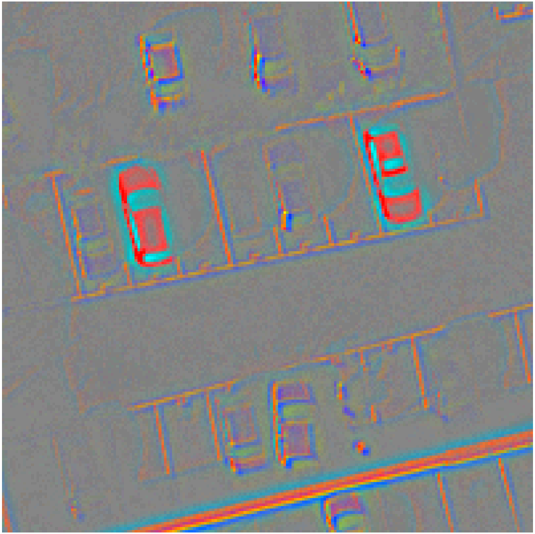
Experimental Results on Simulated Data



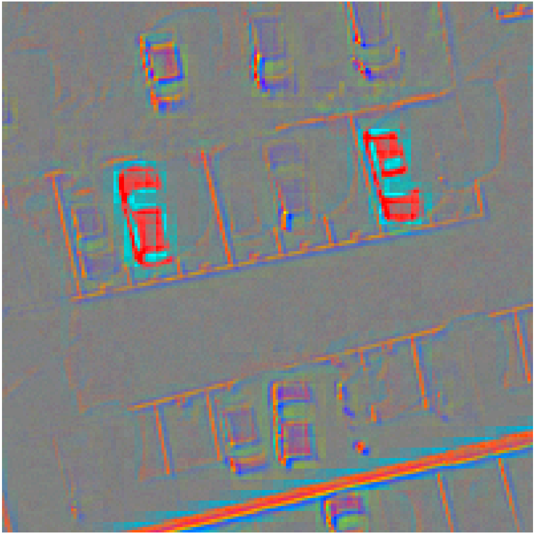
IHS pansharpened image

Proposed Variational Model I

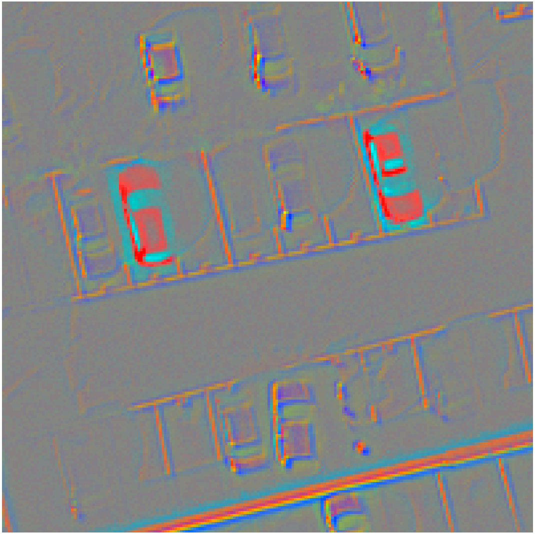
Experimental Results on Simulated Data



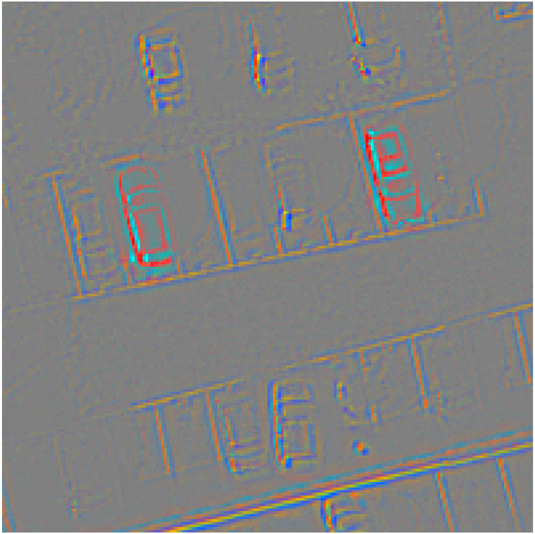
Brovey pansharpened image



Wavelets-based pansharpened image



P+XS pansharpened image



Proposed pansharpened image

Real Satellite Data

- P and XS components not co-registered.
- Forbidden to register/resample XS because of aliasing.
- Panchromatic not a linear combination of MS components.

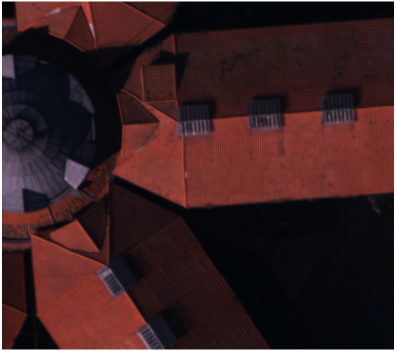
$$\Downarrow$$

Modify proposed functional by decoupling the XS components and avoiding the linearity constraint on the PAN w.r.t. XS components

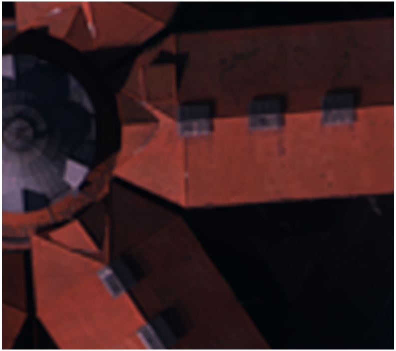
Proposed Variational Model II
Dropping Linearity Constraint

Drop the linearity constraint on PAN w.r.t. XS components:

$$P(x) \equiv \sum_{m=1}^{M} \alpha_m u_m(x), \quad \forall x \in \Omega.$$



Truth image



Result without constraint

Proposed Variational Model II

New Constraint

Introduce the new constraint

$$\frac{u_m(x)}{P(x)} = \frac{\widetilde{u}_m(x)}{\widetilde{P}(x)}, \quad \forall x \in \Omega,$$

where

- P^S $\rightarrow$ PAN at the resolution of S by appropriate downsampling process,
- $\widetilde{P} \rightarrow$ extension of P^S to the whole domain by bicubic interpolation,
- $\widetilde{u} \rightarrow$ extension of $\vec{u}^S$ to the whole domain by bicubic interpolation.

We can rewite the constraint as

$$\underbrace{u_m(x) - \widetilde{u}_m(x)}_{\text{high-frequencies of min}} = \frac{\widetilde{u}_m(x)}{\widetilde{P}(x)} \cdot \underbrace{\left(P(x) - \widetilde{P}(x)\right)}_{\text{high-frequencies of PAN}}, \quad \forall x \in \Omega.$$

In a variational formulation:

$$\sum_{m=1}^{M} \int_{\Omega} \left(u_m(x)\widetilde{P}(x) - \widetilde{u}_m(x)P(x)\right)^2 dx.$$

We propose to minimize the energy

$$
\begin{aligned}
J(\vec{u}) &= \frac{1}{2}\sum_{m=1}^{M}\iint_{\Omega\times\Omega}(u_m(y)-u_m(x))^2\,\omega_P(x,y)\,dy\,dx \\
&+ \frac{\mu s^2}{2}\sum_{m=1}^{M}\int_{\Omega}\Pi_S(x)\left(k_m * u_m(x) - u_m^{\Omega}(x)\right)^2 dx \\
&+ \frac{\delta}{2\|P\|}\sum_{m=1}^{M}\int_{\Omega}\left(u_m(x)\widetilde{P}(x) - \widetilde{u}_m(x)P(x)\right)^2 dx,
\end{aligned}
$$

where $\mu, \delta > 0$ are trade-off parameters.

Proposed Variational Model II

Reformulation of the Energy

Which can be minimized separately for each channel u_m

$$
\begin{aligned}
J(u_m) &= \frac{1}{2}\iint_{\Omega\times\Omega} \left(u_m(y)-u_m(x)\right)^2 \omega_P(x,y)\,dy\,dx \\
&+ \frac{\mu s^2}{2}\int_\Omega \Pi_S(x)\left(k_m * u_m(x) - u_m^\Omega(x)\right)^2 dx \\
&+ \frac{\delta}{2\|P\|}\int_\Omega \left(u_m(x)\widetilde{P}(x) - \widetilde{u}_m(x)P(x)\right)^2 dx,
\end{aligned}
$$

for each $m \in \{1,\dots,M\}$.

We can now proceed as follows:

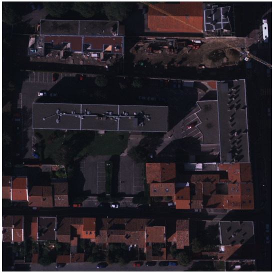
i) Superimpose PAN on each XS component.

ii) Solve the minimization problem for each u_m component independently.

iii) Superimpose all results (free from aliasing) in a common geometry.

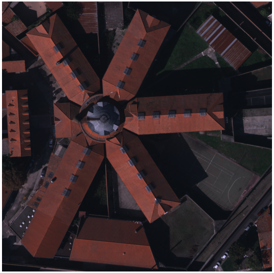
Proposed Variational Model II

Experimental Results on Simulated Data



Houses

Parking



Prison

- Simulate data at 30 cm from full color aerial images:
  - PAN obtained as average of full color components,
  - XS obtained as convolution plus subsampling from full color components.
- Literature model fully satisfied.

Proposed Variational Model II

Experimental Results on Simulated Data

σ	Image	Bicubic	IHS	Brovey	P+XS	NLV I	NLV II
1.7	Houses	8.75	1.98	1.65	1.61	1.34	1.48
	Parking	9.88	1.91	1.58	1.53	1.29	1.41
	Prison	6.56	2.21	1.69	1.62	1.20	1.38
	Avg.	8.40	2.03	1.64	1.59	1.28	1.42
1.5	Houses	8.53	1.92	1.61	1.64	1.36	1.49
	Parking	9.62	1.85	1.54	1.55	1.30	1.41
	Prison	6.36	2.12	1.62	1.66	1.22	1.39
	Avg.	8.17	1.96	1.59	1.62	1.29	1.43
1.3	Houses	8.35	1.88	1.57	1.65	1.40	1.50
	Parking	9.40	1.81	1.50	1.57	1.32	1.42
	Prison	6.18	2.05	1.57	1.67	1.26	1.41
	Avg.	7.98	1.91	1.55	1.63	1.33	1.44
Global Avg.		8.18	1.97	1.59	1.61	1.30	1.43

Full color image

IHS

Brovey

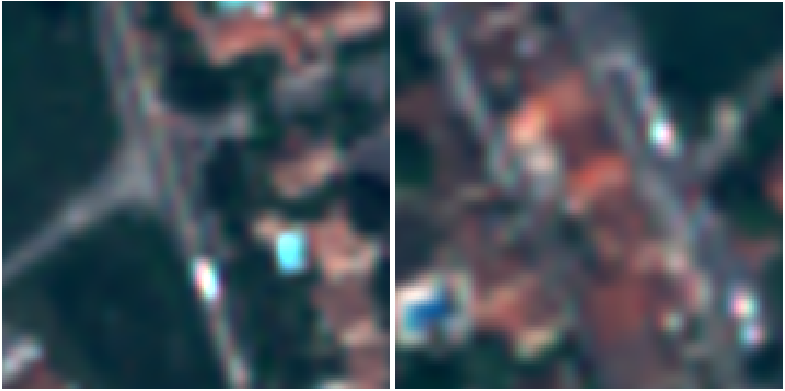
PXS

NLV I

NLV II

- Pléiades produces
  - PAN image at 70 cm per pixel,
  - 4 XS bands (blue, green, red and near-infrared) at 2.8 m per pixel.
- PAN is resampled into the reference of each XS component, thus permitting the pansharpenning of each band separately as proposed for Pléiades images[2].
- Each high-resolution spectral component is obtained from the minimization and then transformed into a common reference for visualization purposes.

[2] Latry, C. and Blanchet, G. and Fourest, S., *Chaine de fusion P+XS Pléiades-HR*, Proc. GRETSI, 2013



Bicubic interpolation on a Toulouse scene captured by Pléiades.

Modified Brovey method on a Toulouse scene captured by Pléiades.

Proposed model on a Toulouse scene captured by Pléiades.