A Novel Framework for Nonlocal Vectorial Total Variation Based on $\ell^{p,q,r}-$ Norms

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- **ROF image denoising model**:

$$\min_{u \in \mathsf{BV}(\Omega)} \mathsf{TV}(u) + \frac{\lambda}{2}\|u - f\|_2^2,$$

with $f \in L^2(\Omega, \mathbb{R})$ being the noisy image and $\lambda > 0$ a trade-off parameter.

- **Color images** $\vec{u} : \Omega \to \mathbb{R}^C$, $\vec{u}(x) = \big(u_1(x), \ldots, u_C(x)\big)$, $x = (x_1, x_2) \in \Omega$:

Vectorial Total Variation
- coupling spatial derivatives
  - isotropic diffusion
  - anisotropic diffusion
- coupling color channels
  - ℓ^1 – coupling
  - ℓ^2 – coupling

- Channel-independent VTV:

$$\sum_{k=1}^{C} \int_{\Omega} \sqrt{\left(\partial_{x_1} u_k(x)\right)^2 + \left(\partial_{x_2} u_k(x)\right)^2} dx$$

- VTV with global coupling:

$$\left(\sum_{k=1}^{C} \left(\int_{\Omega} \sqrt{\left(\partial_{x_1} u_k(x)\right)^2 + \left(\partial_{x_2} u_k(x)\right)^2} \, dx. \right)^2 \right)^{1/2}$$

- Pixel-by-pixel VTV:

$$\int_{\Omega} \left(\sum_{k=1}^{C} \left(\partial_{x_1} u_k(x)\right)^2 + \sum_{k=1}^{C} \left(\partial_{x_2} u_k(x)\right)^2 \right)^{1/2} dx$$

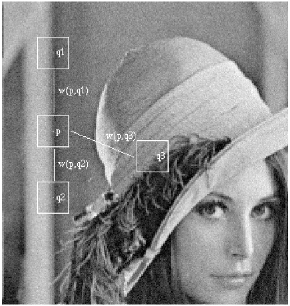
Nonlocal Total Variation

Neighbourhood filters $\rightarrow$ Extend TV to nonlocal regularization:

$$NF[u](x) = \frac{1}{C(x)} \int_\Omega \omega_{u_0}(x,y)u(y)\,dy,$$

where $C(x) = \int_\Omega \omega_{u_0}(x,y)dy$, and $\omega_{u_0} : \Omega \times \Omega \rightarrow \mathbb{R}$ is the **weight distribution.**

Regularity assumption $\rightarrow$ self-similarity



Non-local means filter:

$$\omega_{u_0}(x,y) = e^{-\frac{d_\rho(u_0(x),u_0(y))}{h^2}}$$

$$d_\rho(f(x), f(y)) = \int_\Omega G_\rho(t)|u_0(x+t) - u_0(y+t)|^2 dt.$$

Nonlocal Total Variation

Nonlocal operators for grayscale images $u : \Omega \to \mathbb{R}$

- Nonlocal gradient:

$$\nabla_\omega u(x,y) = \big(u(y) - u(x)\big)\sqrt{\omega(x,y)}, \;\; \forall y \in \Omega.$$

- Nonlocal "isotropic" TV:

$$\int_\Omega \sqrt{\int_\Omega (u(y) - u(x))^2 \, \omega(x,y) \, dy} \, dx.$$

- Nonlocal "anisotropic" TV:

$$\int_\Omega \int_\Omega |u(y) - u(x)| \sqrt{\omega(x,y)} \, dy \, dx.$$

Nonlocal Vectorial TV $\to$ coupling channels with $\ell^2 - norm$:

$$\left(\sum_{k=1}^{C} \left(\int_\Omega \sqrt{\int_\Omega (u_k(y) - u_k(x))^2 \, \omega(x,y) \, dy \, dx} \right)^2 \right)^{1/2}$$

Proposed Framework for VTV

- Color image as $N \times C$ matrix:

$$\mathbf{u} = (u_1, \ldots, u_c) \in \mathbb{R}^{N \times C} \text{ s.t. } u_k \in \mathbb{R}^N, \ \forall k \in \{1, \ldots, C\}.$$

- The Jacobi matrix at each pixel defines a **3D tensor** s.t.

$$K\mathbf{u} \equiv (Ku)_{i,j,k} \in \mathbb{R}^{N \times M \times C}.$$

E.g., $i \in \{1, \ldots, N\}$ corresponds to pixels, $j \in \{1, \ldots, M\}$ to **local or nonlocal derivatives**, and $k \in \{1, \ldots, C\}$ to color channels.

We propose to **regularize 3D tensor with mixed matrix norms**.

Definition

Let $A \in \mathbb{R}^{N \times M \times C}$ be a 3D-matrix. The **mixed matrix** $\ell^{p,q,r}$ **norm** is defined as

$$\|A\|_{p,q,r} = \left(\sum_{i=1}^{N} \left(\sum_{j=1}^{M} \left(\sum_{k=1}^{C} |A_{i,j,k}|^p \right)^{q/p} \right)^{r/q} \right)^{1/r}.$$

Proposed Framework for VTV

Schatten $p-$norms

Compute SVD of the submatrix

$$\begin{pmatrix} \partial_{x_1} u_1(x) & \partial_{x_1} u_2(x) & \partial_{x_1} u_3(x) \\ \partial_{x_2} u_1(x) & \partial_{x_2} u_2(x) & \partial_{x_2} u_3(x) \end{pmatrix} \quad (1)$$

and penalize the singular values with ℓ^p-norm:

- ℓ^1- **Nuclear norm**.
  - Convex relaxation of minimizing the rank of (1).
  - Encourage gradients (jumps) to point into the same direction.
- ℓ^2- **Frobenius norm**.
- **Schatten $\infty-$norm** penalizes the largest singular value.

Definition

Let $A \in \mathbb{R}^{N\times M\times C}$ be a 3D-matrix. The **mixed matrix Schatten (S^p, ℓ^q) norm** as

$$(S^p,\ell^q)(A) = \left(\sum_{i=1}^{N} \left\| \begin{pmatrix} A_{i,1,1} & \cdots & A_{i,1,C} \\ \vdots & \ddots & \vdots \\ A_{i,M,1} & \cdots & A_{i,M,C} \end{pmatrix} \right\|_{S^p}^{q} \right)^{1/q}$$

Mixed matrix norms are not invariant to permutations

$$\swarrow \qquad \searrow$$

$$\ell^{p,q,r}(col, der, pix) \qquad \big(S^p(col, der), \ell^q(pix)\big)$$

Proposed Framework for VTV

Unifying Vectorial Total Variation

Background	Continuous Formulation	Our Framework
Isotropic uncoupled	$\int_\Omega \sum_{k=1}^{C} \sqrt{(\partial_{x_1} u_k(x))^2 + (\partial_{x_2} u_k(x))^2}\, dx$	$\ell^{2,1,1}(der, col, pix)$
Anisotropic uncoupled	$\int_\Omega \sum_{k=1}^{C} \left(\lvert\partial_{x_1} u_k(x)\rvert + \lvert\partial_{x_2} u_k(x)\rvert\right) dx$	$\ell^{1,1,1}(der, col, pix)$
Blomgren Chan	$\sqrt{\sum_{k=1}^{C} \left(\int_\Omega \sqrt{(\partial_{x_1} u_k(x))^2 + (\partial_{x_2} u_k(x))^2}\, dx\right)^2}$	$\ell^{2,1,2}(der, pix, col)$
Anisotropic version	$\sqrt{\sum_{k=1}^{C} \left(\int_\Omega \left(\lvert\partial_{x_1} u_k(x)\rvert + \lvert\partial_{x_2} u_k(x)\rvert\right) dx\right)^2}$	$\ell^{1,1,2}(der, pix, col)$
Bresson Chan	$\int_\Omega \sqrt{\sum_{k=1}^{C} \left(\partial_{x_1} u_k(x)\right)^2 + \sum_{k} \left(\partial_{x_2} u_k(x)\right)^2}\, dx$	$\ell^{2,2,1}(col, der, pix)$
Anisotropic version	$\int_\Omega \sqrt{\sum_{k=1}^{C} \left(\lvert\partial_{x_1} u_k(x)\rvert + \lvert\partial_{x_2} u_k(x)\rvert\right)^2}\, dx$	$\ell^{1,2,1}(der, col, pix)$

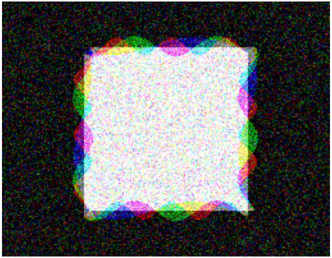
Proposed Framework for VTV

Background	Continuous Formulation	Our Framework
Anisotropic variant	$\int_\Omega \left(\sqrt{\sum_{k=1}^{C} (\partial_{x_1} u_k(x))^2} + \sqrt{\sum_{k=1}^{C} (\partial_{x_2} u_k(x))^2} \right) dx$	$\ell^{2,1,1}(col, der, pix)$
Strong coupling	$\int_\Omega \left(\max_k \|\partial_{x_1} u_k(x)\| + \max_k \|\partial_{x_2} u_k(x)\| \right) dx$	$\ell^{\infty,1,1}(col, der, pix)$
Isotropic version	$\int_\Omega \sqrt{\left(\max_k \|\partial_{x_1} u_k(x)\| \right)^2 + \left(\max_k \|\partial_{x_2} u_k(x)\| \right)^2} dx$	$\ell^{\infty,2,1}(col, der, pix)$
Isotropic variant	$\int_\Omega \max_k \sqrt{(\partial_{x_1} u_k(x))^2 + (\partial_{x_2} u_k(x))^2} dx$	$\ell^{2,\infty,1}(der, col, pix)$
Sapiro	$\int_\Omega \left\| \begin{pmatrix} (\partial_{x_1} u_k(x))_{k=1,\dots,C} \\ (\partial_{x_2} u_k(x))_{k=1,\dots,C} \end{pmatrix} \right\|_{S^1} dx$	$(S^1(col, der), \ell^1(pix))$
Sapiro Goldluecke	$\int_\Omega \left\| \begin{pmatrix} (\partial_{x_1} u_k(x))_{k=1,\dots,C} \\ (\partial_{x_2} u_k(x))_{k=1,\dots,C} \end{pmatrix} \right\|_{S^\infty} dx$	$(S^\infty(col, der), \ell^1(pix))$

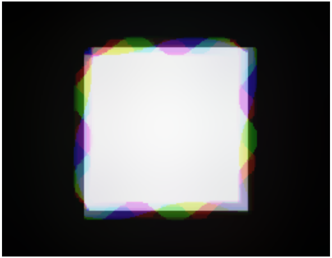
Proposed Framework for VTV

Background	Continuous Formulation	Our Framework
Isotropic uncoupled	$\int_\Omega \left(\sum_{k=1}^{C} \sqrt{\int_\Omega (u_k(y) - u_k(x))^2 \, \omega(x,y) \, dy} \right) \, dx$	$\ell^{2,1,1}(der, col, pix)$
Anisotropic uncoupled	$\int_\Omega \left(\sum_{k=1}^{C} \int_\Omega \lvert u(y) - u(x) \rvert \sqrt{\omega(x,y)} \, dy \right) \, dx$	$\ell^{1,1,1}(der, col, pix)$
Duan Pan, Tai	$\sqrt{\sum_{k=1}^{C} \left(\int_\Omega \sqrt{\int_\Omega (u_k(y) - u_k(x))^2 \, \omega(x,y) \, dy \, dx} \right)^2}$	$\ell^{2,1,2}(der, pix, col)$
Anisotropic coupled	$\int_\Omega \int_\Omega \sqrt{\sum_{k=1}^{C} (u_k(y) - u_k(x))^2 \, \omega(x,y) \, dy \, dx}$	$\ell^{2,1,1}(col, der, pix)$
Isotropic coupled	$\int_\Omega \sqrt{\int_\Omega \sum_{k=1}^{C} (u_k(y) - u_k(x))^2 \, \omega(x,y) \, dy \, dx}$	$\ell^{2,2,1}(col, der, pix)$
Strong coupling	$\int_\Omega \int_\Omega \max_k \left((u_k(y) - u_k(x))^2 \, \omega(x,y) \right) \, dy \, dx$	$\ell^{\infty,1,1}(col, der, pix)$

Which is the best channel coupling?



Noisy

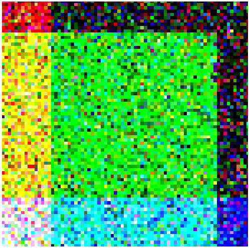


$\ell^{1,1,1}(col, der, pix)$

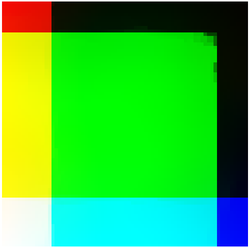
$\ell^{2,1,1}(col, der, pix)$

$\ell^{\infty,1,1}(col, der, pix)$

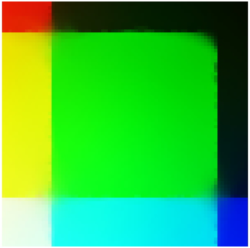
Which is the best channel coupling?



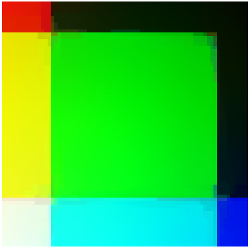
Noisy



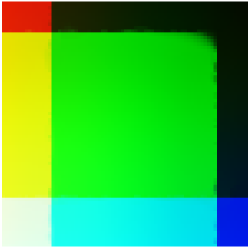
$\ell^{1,1,1}(col, der, pix)$



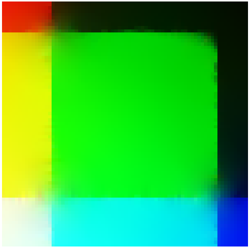
$\ell^{2,2,1}(col, der, pix)$



$\ell^{\infty,1,1}(col, der, pix)$



$(S^1(col, der), \ell^1(pix))$



$(S^\infty(col, der), \ell^1(pix))$

Which is the best channel coupling?

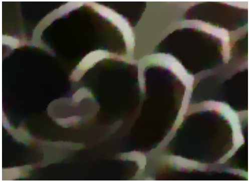
Noisy

$\ell^{1,1,1}(col, der, pix)$

$\ell^{2,2,1}(col, der, pix)$

$\ell^{\infty,1,1}(col, der, pix)$

$(S^1(col, der), \ell^1(pix))$



$(S^\infty(col, der), \ell^1(pix))$

Consider the **saddle-point problem**:

$$\min_{u\in X, g\in Y} \max_{z\in Y} \{G(u) + \langle Ku - g, z\rangle + F(g)\},$$

where $X = \mathbb{R}^{N\times C}$, $Y = \mathbb{R}^{N\times M\times C}$, and $G : X \to \mathbb{R}$ and $F : Y \to \mathbb{R}$ are proper convex l.s.c. functionals. The **optimality conditions** are:

$$-K^T\widehat{z} \in \partial G(\widehat{u}), \quad \widehat{g} \in \partial F^*(\widehat{z}), \quad \widehat{g} = K\widehat{u}.$$

The **Primal-Dual Hybrid Gradient (PDHG)** method computes the solution as

$$\begin{aligned}
u^{n+1} &= \text{prox}_{\tau_n G}\left(u^n - \tau_n K^T z^n\right), && \leftarrow \quad \text{Gradient descent step in } u\\
\bar{u}^{n+1} &= u^{n+1} + \left(u^{n+1} - u^n\right), && \leftarrow \quad \text{Over-relaxation step in } u\\
g^{n+1} &= \text{prox}_{\frac{1}{\sigma_n}F}\left(K\bar{u}^{n+1} + \frac{z^n}{\sigma_n}\right), && \leftarrow \quad \text{Gradient ascent step in } g\\
z^{n+1} &= z^n + \sigma_n\left(K\bar{u}^{n+1} - g^{n+1}\right), && \leftarrow \quad \text{Gradient ascent step in } z
\end{aligned}$$

where $\tau_n, \sigma_n > 0$ are step-size parameters.

Proximal Resolution of the Minimization Problem

The **proximity operator** of F is defined by

$$\text{prox}_{\frac{1}{\sigma}F}(z) = \arg\min_{s\in Y}\left\{\frac{1}{2}\|s-z\|_F^2 + \frac{1}{\sigma}F(s)\right\}.$$

Proximity operators of $\ell^{p,q,r}$ norms

- $\ell^{1,1,1}$**−norm**:

$$\left(\text{prox}_{\frac{1}{\sigma}\|\cdot\|_{1,1,1}}(A)\right)_{i,j,k} = \max\left(|A_{i,j,k}| - \frac{1}{\sigma}, 0\right)\text{sign}(A_{i,j,k}).$$

- $\ell^{2,1,1}$**−norm**:

$$\left(\text{prox}_{\frac{1}{\sigma}\|\cdot\|_{2,1,1}}(A)\right)_{i,j,k} = \max\left(\|A_{i,j,:}\|_2 - \frac{1}{\sigma}, 0\right)\frac{A_{i,j,k}}{\|A_{i,j,:}\|_2},$$

  and similar for $\ell^{2,2,1}$**−norm**.

- $\ell^{\infty,1,1}$**−norm**:

$$\left(\text{prox}_{\frac{1}{\sigma}\|\cdot\|_{\infty,1,1}}(A)\right)_{i,j,k} = A_{i,j,k} - \frac{1}{\sigma}\text{sign}(A_{i,j,k})\left(\text{proj}_{\|\cdot\|_1\leq 1}(\sigma|A_{i,j,:}|)\right)_{i,j,k},$$

  and similar for $\ell^{\infty,\infty,1}$**−norm**.

Proximal Resolution of the Minimization Problem

- $\ell^{\infty,2,1}$**−norm**:

$$\left(\mathsf{prox}_{\frac{1}{\sigma}\|\cdot\|_{\infty,2,1}}(A)\right)_{i,j,k} = A_{i,j,k} - \frac{1}{\sigma}\mathsf{sign}\left(A_{i,j,k}\right)\left(\mathsf{proj}_{\|\cdot\|_{1,2}\leq 1}\left(\sigma|A_{i,:,:}|\right)\right)_{i,j,k}.$$

- $\ell^{2,\infty,1}$**−norm** is computed using the following result:

Theorem

Let $f:\mathbb{R}^{n\times m}\to\mathbb{R}^n$ be $f_i(u) := \sqrt{\sum_{j=1}^m u_{i,j}^2} = \|u_{i,:}\|_2$, and let $g:\mathbb{R}^n\to\mathbb{R}$ be a proper convex function being nondecreasing in each argument. Then

$$\left(\mathit{prox}_{\tau(g\circ f)}(u)\right)_{i,j} = \frac{u_{i,j}}{\|u_{i,:}\|_2}\ \max\left(\|u_{i,:}\|_2 - \tau v_i, 0\right),$$

where the v_i's are the components of the vector $v\in\mathbb{R}^n$ that solves

$$v = \arg\min_{w\in\mathbb{R}^n} \frac{1}{2}\left\|w - \frac{1}{\tau}f(u)\right\|^2 + \frac{1}{\tau}g^*(w).$$

Proximal Resolution of the Minimization Problem

- $\ell^{\infty,2,1}$**−norm**:

$$\left(\text{prox}_{\frac{1}{\sigma}\|\cdot\|_{\infty,2,1}}(A)\right)_{i,j,k} = A_{i,j,k} - \frac{1}{\sigma}\text{sign}\big(A_{i,j,k}\big)\left(\text{proj}_{\|\cdot\|_{1,2}\leq 1}\left(\sigma|A_{i,:,:}|\right)\right)_{i,j,k}.$$

- $\ell^{2,\infty,1}$**−norm**:

$$\left(\text{prox}_{\frac{1}{\sigma}\|\cdot\|_{2,\infty,1}}(A)\right)_{i,j,k} = \frac{A_{i,j,k}}{\|A_{i,j,:}\|_2}\max\left(\|A_{i,j,:}\|_2 - \frac{1}{\sigma}v_{i,j}, 0\right),$$

where $v_{i,j} = \left(\text{prox}_{\|\cdot\|_1\leq 1}\left(\sigma\big(\|A_{i,j,:}\|_2\big)_j\right)\right)_{i,j}$, and $\big(\|A_{i,j,:}\|_2\big)_j$ denotes the vector obtained by stracking $\|A_{i,j,:}\|_2$ for all j.

Experimental Results on Image Denoising

The **image denoising minimization problem** is

$$\min_{u \in \mathbb{R}^{N \times C}} \|Ku\| + \frac{\lambda}{2}\|u - f\|_F^2,$$

where $f \in \mathbb{R}^{N \times C}$ is the noisy image, $\lambda > 0$ the regularization parameter, and $\|\cdot\|$ denotes either an $\ell^{p,q,r}$ norm or a Schatten (S^p, ℓ^q) norm.

The **proximity operator** of $G(u) = \frac{\lambda}{2}\|u - f\|_F^2$ is

$$\mathrm{prox}_{\tau G}(u) = \arg\min_{v \in X} \left\{ \frac{1}{2}\|v - u\|_F^2 + \tau \frac{\lambda}{2}\|v - f\|_F^2 \right\} \Leftrightarrow \mathrm{prox}_{\tau G}(u) = \frac{u + \tau\lambda f}{1 + \tau\lambda}.$$

Influence of the Regularization Parameter

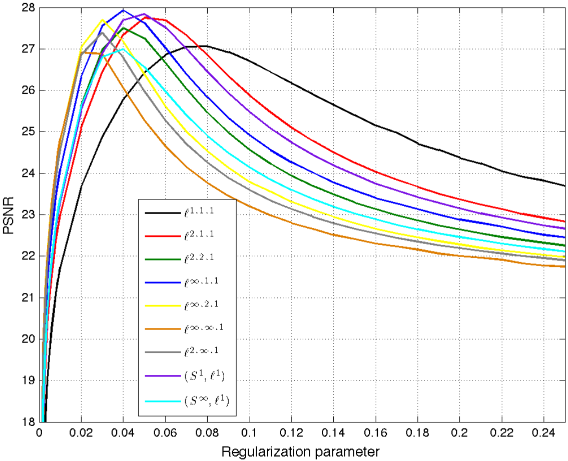


Figure : Comparison of different matrix TV regularizations for color denoising using different λ values on data with Gaussian noise of standard deviation 25.

Experimental Results on Image Denoising

Figure : Noisy image with noise s.t. 25. PSNR $= 20.74$.

Experimental Results on Image Denoising

Figure : $\ell^{\infty,1,1}-$regularization with $\lambda = 0.1$. PSNR $= 24.92$.

Experimental Results on Image Denoising

Figure : $\ell^{\infty,1,1}-$regularization with $\lambda = 0.05$. PSNR $= 27.62$.

Experimental Results on Image Denoising

Figure : $\ell^{\infty,1,1}-$regularization with optimal $\lambda = 0.04$. PSNR $= 27.93$.

Experimental Results on Image Denoising

Figure : $\ell^{\infty,1,1}-$regularization with $\lambda = 0.03$. PSNR $= 27.55$.

Experimental Results on Image Denoising

Figure : $\ell^{\infty,1,1}-$regularization with $\lambda = 0.01$. PSNR $= 24.09$.

Kodak Database


Kodak 1


Kodak 2


Kodak 3


Kodak 4


Kodak 5


Kodak 6


Kodak 7


Kodak 8


Kodak 9


Kodak 10


Kodak 11


Kodak 12

Experimental Results on Image Denoising

Nonlocal Color TV Denoising

Kodak	Noisy	$\ell^{1,1,1}$	$\ell^{2,1,1}$	$\ell^{2,2,1}$	$\ell^{\infty,1,1}$
1	26.15	31.01	31.14	31.07	**31.20**
2	26.14	31.23	31.36	31.21	**31.44**
3	26.17	31.78	31.88	31.76	**31.99**
4	26.08	34.38	**35.06**	34.66	35.03
5	26.10	35.02	35.69	35.35	**35.73**
6	26.11	29.28	29.37	29.30	**29.60**
7	26.08	31.64	31.70	31.58	**31.77**
8	26.31	33.88	34.24	34.02	**34.29**
9	26.98	34.40	34.74	34.67	**34.78**
10	26.06	32.21	32.50	32.36	**32.61**
11	26.06	32.31	32.39	32.27	**32.45**
12	26.09	35.17	35.93	35.33	**35.94**
Avg.	26.19	32.69	33.00	32.80	**33.07**

Table : For each matrix TV method, the optimal λ and h in terms of PSNR were computed on the first Kodak image and then used on the others. The input noise s.d. was 12.75.

Experimental Results on Image Denoising

Figure : Clean image.

Experimental Results on Image Denoising

Figure : Noisy image with noise s.d. 12.75. PSNR $= 26.09$.

Experimental Results on Image Denoising

Figure : $\ell^{1,1,1}$−NLTV regularization. PSNR $= 35.17$.

Experimental Results on Image Denoising

Figure : $\ell^{2,1,1}-$NLTV regularization. PSNR $= 35.93$.

Figure : $\ell^{2,2,1}-$NLTV regularization. PSNR $= 35.33$.

Experimental Results on Image Denoising

Figure : $\ell^{\infty,1,1}$−NLTV regularization. PSNR = 35.94.

Experimental Results on Image Denoising

Figure : Clean image.

Experimental Results on Image Denoising

Figure : Noisy image with noise s.d. 12.75. PSNR $= 26.10$.

Figure : $\ell^{1,1,1}-$TV regularization. PSNR $= 33.60$.

Experimental Results on Image Denoising

Figure : $\ell^{1,1,1}$−NLTV regularization. PSNR $= 35.41$.

Experimental Results on Image Denoising

Figure : $\ell^{\infty,1,1}-$TV regularization. PSNR $= 34.88$.

Experimental Results on Image Denoising

Figure : $\ell^{\infty,1,1}$ −NLTV regularization. PSNR $= 35.65$.

Local Color TV Denoising

	Noisy	$\ell^{1,1,1}$	$\ell^{2,1,1}$	$\ell^{2,2,1}$	$\ell^{\infty,1,1}$	$\ell^{\infty,2,1}$	$\ell^{2,\infty,1}$	(S^1, ℓ^1)	(S^∞, ℓ^1)
1	24.78	28.14	29.07	28.51	**29.90**	29.19	29.07	29.20	27.96
2	24.76	28.54	29.48	29.22	**30.18**	29.87	29.66	29.83	28.62
3	24.80	29.20	30.15	29.81	**30.85**	30.51	30.25	30.33	29.24
4	24.68	30.92	32.22	31.80	**32.73**	32.71	32.13	32.32	31.01
5	24.71	31.50	32.75	32.41	33.13	**33.30**	32.64	32.81	31.65
6	24.72	27.36	28.19	27.98	**29.01**	28.64	28.52	28.59	27.47
7	24.71	29.46	30.39	30.12	**30.86**	30.71	30.35	30.57	29.53
8	24.96	31.08	32.10	31.84	**32.41**	32.40	32.02	32.20	31.22
9	25.68	30.92	31.74	31.54	**32.10**	32.00	31.78	31.85	31.11
10	24.66	29.75	30.81	30.49	**31.48**	31.29	30.94	31.05	29.84
11	24.66	30.14	31.10	30.84	**31.49**	31.46	31.07	31.22	30.25
12	24.71	31.85	33.15	32.84	33.45	**33.69**	33.03	33.25	32.05
	24.82	29.91	30.93	30.62	**31.47**	31.31	30.96	31.10	30.00

Table : For each matrix TV method, the optimal λ in terms of PSNR was computed on the first Kodak image and then used on the others. The input noise s.d. was 15.

Figure : Clean image.

Figure : Noise image with noise s.d. 15. PSNR $= 24.78$

Experimental Results on Image Denoising

Figure : $\ell^{1,1,1}-$TV regularization. PSNR $= 28.14$.

Experimental Results on Image Denoising

Figure : $\ell^{2,1,1}-$TV regularization. PSNR $= 29.07$.

Experimental Results on Image Denoising

Figure : $\ell^{2,2,1}-$TV regularization. PSNR $= 28.51$.

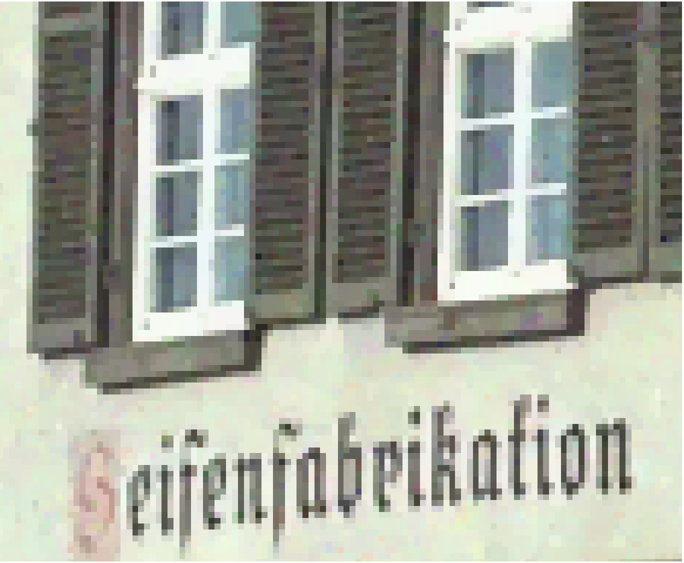


Figure : $\ell^{\infty,1,1}$−TV regularization. PSNR $= 29.90$.

Experimental Results on Image Denoising

Figure : $\ell^{\infty,2,1}-$TV regularization. PSNR $= 29.19$.

Experimental Results on Image Denoising

Figure : $\ell^{2,\infty,1}-$TV regularization. PSNR $= 29.07$.

Figure : $(S^1, \ell^1)-$TV regularization. PSNR $= 29.20$.

Experimental Results on Image Denoising

Figure : $(S^\infty, \ell^1)-$TV regularization. PSNR $= 27.96$.

Conclusions and Future Work

- We proposed a novel framework that unifies vectorial TV regularizations.
- The gradient of a color image was interpreted as a 3D tensor using pixels, (local or nonlocal) derivatives, and color channels.
- We considered several $\ell^{p,q,r}$ and (S^p, ℓ^q) matrix norms.
- Based on our experiments, we exhibited the superiority of using ℓ^∞ inter-channel coupling for a stronger suppression of color artifacts in natural images.
- Future work will mainly concentrate on:
  - development of more mixed matrix norms for TV and NLTV regularizations;
  - study of permutations in the dimensions of the 3D tensor;
  - application of color transformations;
  - mathematical properties of mixed matrix norms;
  - application to other image restoration problems.

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