

# Nonlocal Regularizing Constraints in Variational Optical Flow

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**J. Duran** and A. Buades

joan.duran@uib.es, toni.buades@uib.es

Dept. Mathematics and Computer Science  
University of Balearic Islands, Mallorca, Spain

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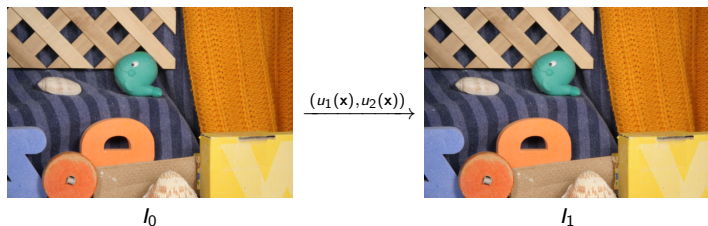
**Universitat**  
de les Illes Balears



**Obra Social "la Caixa"**

# Introduction

- ▶ Optical Flow → compute a correspondence field between an image pair to capture the apparent dynamical behaviour of the objects in the scene.



- ▶ Classification of methods

- **Local** → point matching ⇒ **sparse flow**.

★ Lucas-Kanade<sup>1</sup> model:  $K_\rho * (I_x u_1 + I_y u_2 + I_t)^2$

- **Global/variational** → regularized energy minimization ⇒ **dense flow**.

★ Horn-Schunck<sup>2</sup> model:  $\int_{\Omega} (I_x u_1 + I_y u_2 + I_t)^2 dx + \lambda \int_{\Omega} (|\nabla u_1|^2 + |\nabla u_2|^2) dx$

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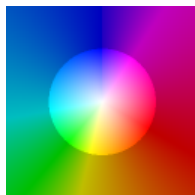
<sup>2</sup>B. Horn and B. Schunck, Determining optical flow, Proc. Tech. Symp. East, Int. Society for Optics and Photonics, pp. 319-331, 1981.

# Introduction

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Flow



Color coding

- ▶ Classification of methods

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## Notations

- ▶  $\Omega$  rectangular domain in  $\mathbb{R}^2$ .
- ▶  $I : \Omega \times [0, T] \rightarrow \mathbb{R}$  image sequence.
- ▶  $I(\mathbf{x}, t)$  intensity at pixel  $\mathbf{x} = (x, y) \in \Omega$  and time  $0 \leq t \leq T$ .
- ▶  $\mathbf{u} : \Omega \times [0, T] \rightarrow \mathbb{R}^2$  flow field,  $\mathbf{u}(\mathbf{x}, t) = (u_1(\mathbf{x}, t), u_2(\mathbf{x}, t))$ .
- ▶ Drop dependency of variables over  $t$ , so  $I_0(\mathbf{x}) = I(\mathbf{x}, t)$  and  $I_1(\mathbf{x}) = I(\mathbf{x}, t + 1)$ .

## Variational framework

$$\min_{\mathbf{u}} E(\mathbf{u}) = E_d(\mathbf{u}) + \lambda E_r(\mathbf{u})$$

s.t.  $\mathbf{u}$  with low energy  $\Leftrightarrow \mathbf{u}$  satisfying desired properties.

## Brightness Constancy Assumption (BCA)

$$I(\mathbf{x} + \mathbf{u}(\mathbf{x}, t), t + 1) - I(\mathbf{x}, t) = 0, \quad \forall \mathbf{x} \in \Omega$$

- ▶ Challenges → **nonlinearity** in  $I(\mathbf{x} + \mathbf{u}(\mathbf{x}, t), t + 1)$ .
- ▶ **Optical flow constraint (OFC)**:

$$\nabla I(\mathbf{x}, t) \cdot \mathbf{u}(\mathbf{x}, t) + I_t(\mathbf{x}, t) = 0, \quad \forall \mathbf{x} \in \Omega$$

**Only valid for small displacements or very smooth images!**

For large displacements:

- Embed minimization in a coarse-to-fine warping<sup>3</sup>.
  - Postpone any linearization to numerical scheme<sup>4</sup>.
- ▶ Shortcomings → **sensitive to additive illumination changes** in the scene.

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<sup>3</sup>M. Black and P. Anandan, The robust estimation of multiple motions: Parametric and piecewise smooth flow fields, *Comput. Vis. Image Underst.*, vol. 63(1), pp. 75-104, 1996.

<sup>4</sup>T. Brox, A. Bruhn, N. Papenberg and J. Weickert, High accuracy optical flow estimation based on a theory for warping, *Proc. ECCV, LNCS vol. 3024*, pp. 25-36, 2004.

### Gradient Constancy Assumption (GCA)<sup>5</sup>

$$\nabla I(\mathbf{x} + \mathbf{u}(\mathbf{x}, t), t + 1) - \nabla I(\mathbf{x}, t) = 0, \quad \forall \mathbf{x} \in \Omega$$

- ▶ Benefits → robust to additive illumination changes.
- ▶ Shortcomings w.r.t. BCA:
  - More **sensitive to noise**.
  - Performing poorly in smooth regions.



**combine BCA & GCA as data-fidelity term**

- ▶ Higher-order constancy conditions much more sensitive to noise than GCA.

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<sup>5</sup>T. Brox, A. Bruhn, N. Papenberg and J. Weickert, High accuracy optical flow estimation based on a theory for warping, Proc. ECCV, LNCS vol. 3024, pp. 25-36, 2004.

### Window Regularized Constraints $\rightarrow$ flow constant over each neighbourhood

- ▶ Integrate local information by **regularizing BCA isotropically**<sup>6</sup>:

$$\int_{\Omega} K_{\rho}(\mathbf{x} - \mathbf{y}) \psi (|I_1(\mathbf{y} + \mathbf{u}(\mathbf{x})) - I_0(\mathbf{y})|) d\mathbf{y}, \quad \forall \mathbf{x} \in \Omega$$

- Benefits  $\rightarrow$  robust to very high noise.
- Shortcomings  $\rightarrow$  **blur motion discontinuities!**

- ▶ **Truncated Normalized cross correlation**<sup>7</sup>:

$$\min \left\{ 1, 1 - \int_{\mathcal{N}(\mathbf{x})} \frac{(I_0(\mathbf{y}) - \mu_0(\mathbf{x}))}{\sigma_0(\mathbf{x})} \cdot \frac{(I_1(\mathbf{y} + \mathbf{u}(\mathbf{x})) - \mu_1(\mathbf{x} + \mathbf{u}(\mathbf{x})))}{\sigma_1(\mathbf{x} + \mathbf{u}(\mathbf{x}))} d\mathbf{y} \right\}, \quad \forall \mathbf{x} \in \Omega$$

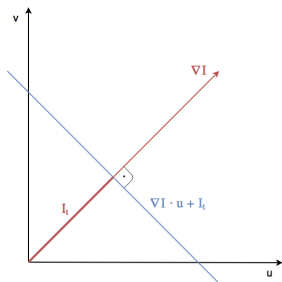
- Disregard negative correlations to gain robustness against occlusions.
- Benefits  $\rightarrow$  Robust to multiplicative and linear illumination changes.
- Shortcomings  $\rightarrow$  **Highly nonlinear!**

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<sup>6</sup>A. Bruhn, J. Weickert and C. Schnörr, Lucas/Kanade meets Horn/Schunck: Combining local and global optic flow methods, Int. J. Comput. Vis., vol. 61(3), pp. 211-231, 2005.

<sup>7</sup>M. Werlberger, T. Pock and H. Bischof, Motion estimation with non-local total variation regularization, CVPR, pp. 464-471, 2010.

## Aperture problem



- ▶  $\text{OFC} \perp \nabla I \Rightarrow \mathbf{u}(\mathbf{x}) = -I_t(\mathbf{x}) \frac{\nabla I(\mathbf{x})}{|\nabla I(\mathbf{x})|^2}$  if  $\nabla I(\mathbf{x}) \neq \mathbf{0}$ .
- ▶ Data constraints not sufficient to uniquely estimate  $\mathbf{u} \Rightarrow$  **ill-posed inverse problem!**
- ▶ Regularization  $\rightarrow$  smoothness in regions of coherent motion while preserving flow discontinuities at boundaries of moving objects.



► **Total variation regularization:**

$$\int_{\Omega} (|\nabla u_1| + |\nabla u_2|) dx$$

- Shortcomings → **staircasing effect, rounded and dislocated contours!**

► **Nonlocal regularization:**

$$\int_{\Omega} \int_{\mathcal{N}(\mathbf{x})} \omega(\mathbf{x}, \mathbf{y}) \cdot \phi(\mathbf{u}(\mathbf{y}) - \mathbf{u}(\mathbf{x})) dy dx$$

- Use coherence of neighbouring pixels to enforce similar motion patterns.
- Support weights based on spatial closeness and intensity similarity:

$$\omega(\mathbf{x}, \mathbf{y}) = \exp\left(-\frac{\|\mathbf{x} - \mathbf{y}\|^2}{h_s^2} - \frac{\|I_0(\mathbf{x}) - I_0(\mathbf{y})\|^2}{h_c^2}\right)$$

- Shortcomings → **copy image details into flow!**

# Nonlocal Regularizing Optical Flow Constraints

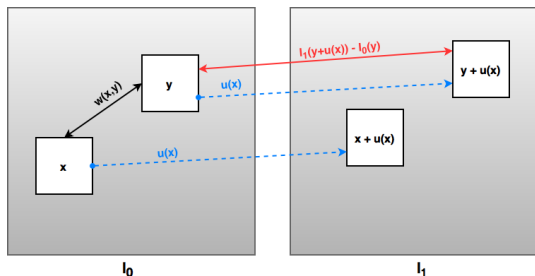
- ▶ We propose two new nonlocal data-fidelity terms.
- ▶ Nonlocal similarity measures restricted to regularization term so far.
- ▶ We use **nonlocal similarity configurations in optical flow constraints**:
  - Image geometry used to regularize the flow and locate flow discontinuities.
  - Motion patterns enforced through coherence of similar pixels.
  - Similarity measure  $\rightarrow$  patch comparison.
- ▶ Main goal  $\rightarrow$  compare performance of proposed data terms w.r.t. to BCA.

# Nonlocal Regularizing Optical Flow Constraints

Nonlocal Brightness Constancy Assumption (NLBCA)

$$E_{\gamma}(\mathbf{u}) = \int_{\Omega} \int_{\Omega} \omega(\mathbf{x}, \mathbf{y}, I_0(\mathbf{x}), I_0(\mathbf{y})) \cdot \psi(|I_1(\mathbf{y} + \mathbf{u}(\mathbf{x})) - I_0(\mathbf{y})|) d\mathbf{y} d\mathbf{x}$$

$$\omega(\mathbf{x}, \mathbf{y}, I_0(\mathbf{x}), I_0(\mathbf{y})) = \frac{1}{\Gamma(\mathbf{x})} \cdot \exp\left(-\frac{\|\mathbf{x} - \mathbf{y}\|^2}{h_s^2}\right) \cdot \exp\left(-\frac{d_{\rho}(I_0(\mathbf{x}), I_0(\mathbf{y}))}{h_c^2}\right)$$



- ▶ Regularizes BCA  $\rightarrow$  coherent motion of similarly appearing neighborhoods.
- ▶ Assumption  $\rightarrow$  **close pixels with similar patch configuration have similar flow.**
- ▶ Bilateral weight distribution<sup>8</sup>.

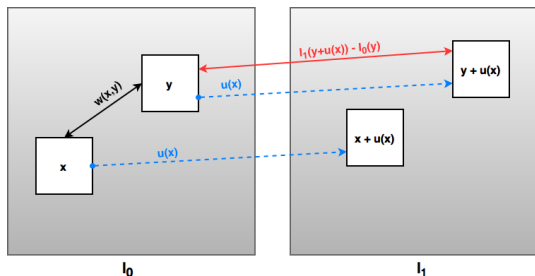
<sup>8</sup>K. Yoon and I. Kweon, Adaptive support-weight approach for correspondence search, IEEE Trans. Pattern Anal. Mach. Intell., vol. 28(4), pp. 650-656, 2006.

# Nonlocal Regularizing Optical Flow Constraints

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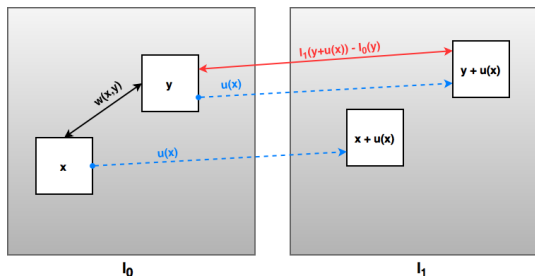
- ▶ Softer than NL regularization, which imposes image details into flow.
- ▶ Advantages w.r.t. isotropic window regularizing constraints → avoids blurring of flow close to motion discontinuities while regularizing it.

# Nonlocal Regularizing Optical Flow Constraints

Nonlocal Brightness Constancy Assumption (NLBCA)

$$E_{\gamma}(\mathbf{u}) = \int_{\Omega} \int_{\Omega} \omega(\mathbf{x}, \mathbf{y}, I_0(\mathbf{x}), I_0(\mathbf{y})) \cdot \psi(|I_1(\mathbf{y} + \mathbf{u}(\mathbf{x})) - I_0(\mathbf{y})|) \, d\mathbf{y} \, d\mathbf{x}$$

$$\omega(\mathbf{x}, \mathbf{y}, I_0(\mathbf{x}), I_0(\mathbf{y})) = \frac{1}{\Gamma(\mathbf{x})} \cdot \exp\left(-\frac{\|\mathbf{x} - \mathbf{y}\|^2}{h_s^2}\right) \cdot \exp\left(-\frac{d_{\rho}(I_0(\mathbf{x}), I_0(\mathbf{y}))}{h_c^2}\right)$$



- ▶ Shortcomings → **pixels with similar patch configuration having different motion!**

Weights allows NLBCA matching:

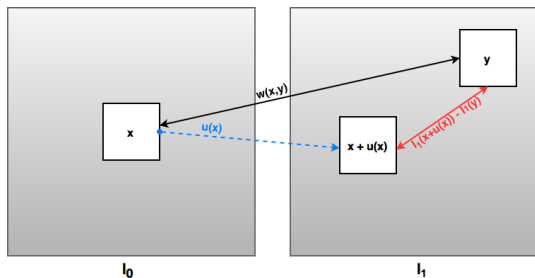
- for spatially close pixels,
- for pixels sharing the intensity of a whole patch and not only pixel intensity.

# Nonlocal Regularizing Optical Flow Constraints

Nonlocal Matching Assumption (NLMA)

$$E_{\delta}(\mathbf{u}) = \int_{\Omega} \int_{\Omega} \omega(l_0(\mathbf{x}), l_1(\mathbf{y})) \cdot \psi(|l_1(\mathbf{x} + \mathbf{u}(\mathbf{x})) - l_1(\mathbf{y})|) \, d\mathbf{y} \, d\mathbf{x}$$

$$\omega(l_0(\mathbf{x}), l_1(\mathbf{y})) = \frac{1}{\Gamma(\mathbf{x})} \cdot \exp\left(-\frac{d_{\rho}(l_0(\mathbf{x}), l_1(\mathbf{y}))}{h_c^2}\right)$$



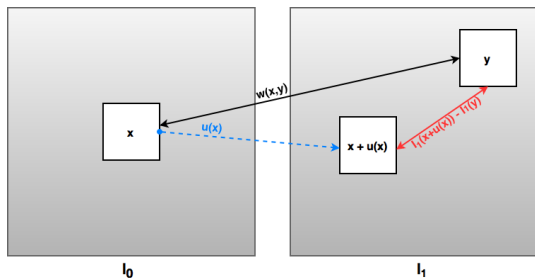
- ▶ Replaces BCA → NLMA no longer impose a constraint on motion trajectories.
- ▶ Assumption → **cross-frame patch similarity preserved by flow.**
- ▶ Weights not depending on spatial closeness.

# Nonlocal Regularizing Optical Flow Constraints

Nonlocal Matching Assumption (NLMA)

$$E_{\delta}(\mathbf{u}) = \int_{\Omega} \int_{\Omega} \omega(l_0(\mathbf{x}), l_1(\mathbf{y})) \cdot \psi(|l_1(\mathbf{x} + \mathbf{u}(\mathbf{x})) - l_1(\mathbf{y})|) \, d\mathbf{y} \, d\mathbf{x}$$

$$\omega(l_0(\mathbf{x}), l_1(\mathbf{y})) = \frac{1}{\Gamma(\mathbf{x})} \cdot \exp\left(-\frac{d_{\rho}(l_0(\mathbf{x}), l_1(\mathbf{y}))}{h_c^2}\right)$$



- ▶ Cross-frame weights  $\rightarrow$  combine optical flow and block matching techniques.
- ▶ Regularity of warped image  $\rightarrow$  artifact suppression due to wrong flows and noise.
- ▶ Shortcomings  $\rightarrow$  artifacts due to flow may not be more prominent than noise!

# Nonlocal Regularizing Optical Flow Constraints

## Energy Functionals

- ▶ Penalize deviations from both constraints with **quadratic functions**  $\rightarrow \psi(s) = s^2$ .
- ▶ Spatial coherence of flow through **TV regularization**.
- ▶ **Linearize both constraints** using first-order Taylor expansions.
- ▶ Final linearized energies for optical flow estimation:

- **NLBCA based linearized energy**

$$E_{\gamma}^l(\mathbf{u}) := \int_{\Omega} (|\nabla u_1(\mathbf{x})| + |\nabla u_2(\mathbf{x})|) d\mathbf{x} \\ + \frac{\gamma}{2} \iint_{\Omega \times \Omega} \omega(\mathbf{x}, \mathbf{y}, l_0(\mathbf{x}), l_0(\mathbf{y})) (l_1(\mathbf{y} + \mathbf{u}^0(\mathbf{x})) - l_0(\mathbf{y}) + \langle \nabla l_1(\mathbf{y} + \mathbf{u}^0(\mathbf{x})), \mathbf{u}(\mathbf{x}) - \mathbf{u}^0(\mathbf{x}) \rangle)^2 d\mathbf{y} d\mathbf{x}$$

- **NLMA based linearized energy**

$$E_{\delta}^l(\mathbf{u}) := \int_{\Omega} (|\nabla u_1(\mathbf{x})| + |\nabla u_2(\mathbf{x})|) d\mathbf{x} \\ + \frac{\delta}{2} \iint_{\Omega \times \Omega} \omega(l_0(\mathbf{x}), l_1(\mathbf{y})) (l_1(\mathbf{x} + \mathbf{u}^0(\mathbf{x})) - l_1(\mathbf{y}) + \langle \nabla l_1(\mathbf{x} + \mathbf{u}^0(\mathbf{x})), \mathbf{u}(\mathbf{x}) - \mathbf{u}^0(\mathbf{x}) \rangle)^2 d\mathbf{y} d\mathbf{x}$$



# Numerical Minimization

## Convex Relaxation of the Energies

- ▶ **Convex relaxation** → decoupling energy terms using an auxiliary variable:

$$E_{\gamma}^l(\mathbf{u}) := \int_{\Omega} (|\nabla u_1(\mathbf{x})| + |\nabla u_2(\mathbf{x})|) d\mathbf{x} + \frac{1}{2\theta} \|\mathbf{u} - \mathbf{v}\|^2 \\ + \frac{\gamma}{2} \iint_{\Omega \times \Omega} \omega(\mathbf{x}, \mathbf{y}, l_0(\mathbf{x}), l_0(\mathbf{y})) (h_1(\mathbf{y} + \mathbf{u}^0(\mathbf{x})) - l_0(\mathbf{y}) + \langle \nabla h_1(\mathbf{y} + \mathbf{u}^0(\mathbf{x})), \mathbf{v}(\mathbf{x}) - \mathbf{u}^0(\mathbf{x}) \rangle)^2 d\mathbf{y}d\mathbf{x}$$

$$E_{\delta}^l(\mathbf{u}) := \int_{\Omega} (|\nabla u_1(\mathbf{x})| + |\nabla u_2(\mathbf{x})|) d\mathbf{x} + \frac{1}{2\theta} \|\mathbf{u} - \mathbf{v}\|^2 \\ + \frac{\delta}{2} \iint_{\Omega \times \Omega} \omega(l_0(\mathbf{x}), l_1(\mathbf{y})) (h_1(\mathbf{x} + \mathbf{u}^0(\mathbf{x})) - l_1(\mathbf{y}) + \langle \nabla h_1(\mathbf{x} + \mathbf{u}^0(\mathbf{x})), \mathbf{v}(\mathbf{x}) - \mathbf{u}^0(\mathbf{x}) \rangle)^2 d\mathbf{y}d\mathbf{x}$$

- ▶ **Alternate minimizations:**

- Fixed  $\mathbf{v}$ , solve TV-based problem using Chambolle's projection algorithm.
- Fixed  $\mathbf{u}$ , compute explicit solution in  $\mathbf{v}$  using Euler-Lagrange equation.

# Numerical Minimization

## Computation of Weight Distributions

**Nonlocal interaction limited to pixels at a certain distance:**

$$\omega(\mathbf{x}, \mathbf{y}, l_0(\mathbf{x}), l_0(\mathbf{y})) = \begin{cases} \frac{1}{\Gamma(\mathbf{x})} \exp\left(-\frac{\|\mathbf{x} - \mathbf{y}\|^2}{h_s^2}\right) \exp\left(-\sum_{\mathbf{z} \in \mathcal{N}_0} \frac{|l_0(\mathbf{x} + \mathbf{z}) - l_0(\mathbf{y} + \mathbf{z})|^2}{h_c^2}\right) & \text{if } \|\mathbf{x} - \mathbf{y}\|_\infty \leq \nu \\ 0 & \text{otherwise} \end{cases}$$

$$\omega(l_0(\mathbf{x}), l_1(\mathbf{y})) = \begin{cases} \frac{1}{\Gamma(\mathbf{x})} \exp\left(-\sum_{\mathbf{z} \in \mathcal{N}_0} \frac{|l_0(\mathbf{x} + \mathbf{z}) - l_1(\mathbf{y} + \mathbf{z})|^2}{h_c^2}\right) & \text{if } \|\mathbf{x} - \mathbf{y}\|_\infty \leq \nu \\ 0 & \text{otherwise} \end{cases}$$

- $\nu > 0$  prescribed parameter (research window).
- $\mathcal{N}_0$  is a rectangular window centered at origin (comparison window).
- In practice  $\rightarrow 21 \times 21$  research window and  $7 \times 7$  comparison window.

# Numerical Minimization

## Multiscale Approach

- ▶ Coarse-to-fine scheme to reduce distance between objects:
  - 5-scale image pyramid with sampling factor 2.
  - Energy minimization at each scale and propagate flow as  $\mathbf{u}^{s-1}(\mathbf{x}) = 2 \cdot \mathbf{u}^s(\frac{\mathbf{x}}{2})$ .
  - Intermediate solution used as initialization in following scale.
  - 5-warping steps to refine  $\mathbf{u}^0$  and  $I_1(\cdot + \mathbf{u}^0)$  at each scale.
- ▶ Flow computed on grayscale images.
- ▶ Use  $7 \times 7$  median filter to increase robustness w.r.t sampling artifacts in data<sup>9</sup>.
- ▶ Drawback → motion of small objects undergoing large displacements cannot be estimated since may disappear in the coarsest scales!

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<sup>9</sup>A. Wedel, T. Pock, C. Zach, D. Cremers and H. Bischof, An improved algorithm for TV-L1 optical flow, Proc. Statistical and Geometrical Approached to Visual Motion Analysis, LNCS vol. 5604, pp. 23-45, 2009.

## Experimental Results

- ▶ Evaluation on Middlebury benchmark<sup>10</sup> with known ground truth.
- ▶ Optimal trade-off parameters in terms of lowest average end-point error (AEPE):

$$\text{AEPE}(\mathbf{u}) = \frac{1}{|\Omega|} \int_{\Omega} |\mathbf{u}(\mathbf{x}) - \mathbf{u}^{\text{ref}}(\mathbf{x})| d\mathbf{x}$$

- ▶ Exclude pixels in the occlusions, which are available for the Middlebury benchmark, when computing AEPE.
- ▶ Computational cost of NLBCA and NLMA equivalent to classical BCA → extra weight computation at the beginning of each scale might be easily parallelised!

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<sup>10</sup>S. Barker, D. Scharstein, J. Lewis, S. Roth, M. Black and R. Szeliski, A database and evaluation methodology for optical flow, *Int. J. Comput. Vis.*, vol. 92(1), pp. 1-31, 2011.

# Experimental Results

## Comparison of Data Constraints on Venus Sequence



10

# Experimental Results

## Comparison of Data Constraints on Venus Sequence



$I_1$

# Experimental Results

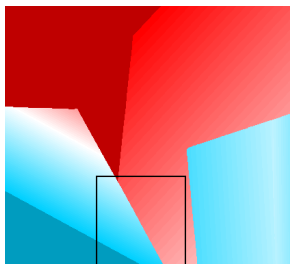
Comparison of Data Constraints on Venus Sequence



Ground-truth flow

# Experimental Results

Comparison of Data Constraints on Venus Sequence



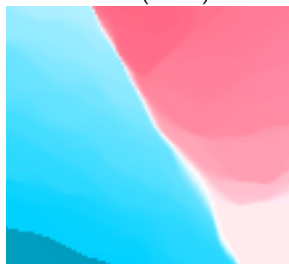
Reference



BCA (0.313)



NLBCA (0.309)

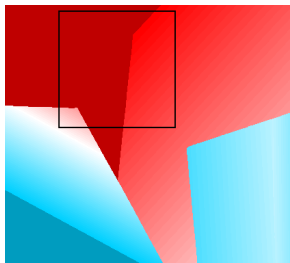


NLMA (0.310)



# Experimental Results

Comparison of Data Constraints on Venus Sequence



Reference



BCA (0.313)



NLBCA (0.309)



NLMA (0.310)

# Experimental Results

Comparison of Data Constraints on Rubberwhale Sequence



$I_0$

# Experimental Results

Comparison of Data Constraints on Rubberwhale Sequence



$h_1$

# Experimental Results

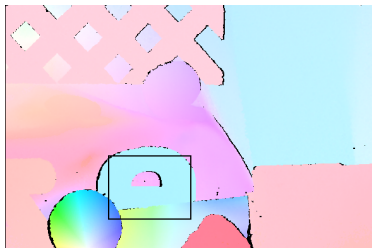
Comparison of Data Constraints on Rubberwhale Sequence



Ground-truth flow

# Experimental Results

Comparison of Data Constraints on Rubberwhale Sequence



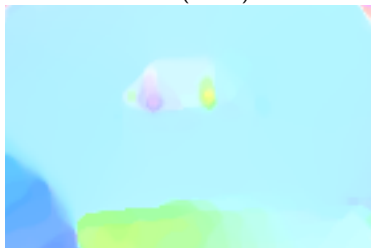
Reference



BCA (0.209)



NLBCA (0.154)



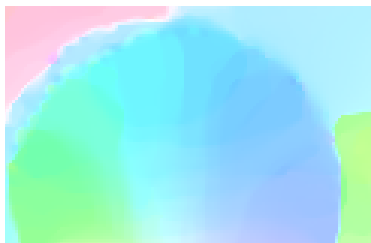
NLMA (0.199)

# Experimental Results

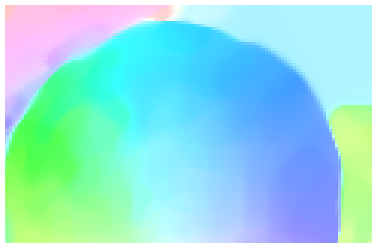
Comparison of Data Constraints on Rubberwhale Sequence



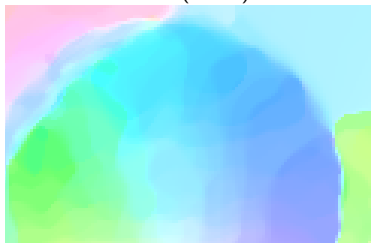
Reference



BCA (0.209)



NLBCA (0.154)



NLMA (0.199)

# Experimental Results

Comparison of Data Regularizing Constraints with Nonlocal Regularization on Venus Sequence



$I_0$

# Experimental Results

Comparison of Data Regularizing Constraints with Nonlocal Regularization on Venus Sequence



$I_1$



# Experimental Results

Comparison of Data Regularizing Constraints with Nonlocal Regularization on Venus Sequence



Ground-truth flow

# Experimental Results

Comparison of Data Regularizing Constraints with Nonlocal Regularization on Venus Sequence



Reference



Nonlocal regularization



NLBCA



NLMA

# Conclusions

- ▶ We have introduced two nonlocal regularizing constraints for variational optical flow estimation.
- ▶ Preliminary results illustrate
  - superiority of NLBCA w.r.t. classical BCA,
  - similar performances of NLMA and BCA while being completely different,
  - image self-similarity can be better taken advantage of in the data-fidelity terms rather than in the regularization prior.
- ▶ Limitations are in the optimization strategy rather than in the models themselves.
- ▶ Future work:
  - Exhaustive performance comparison.
  - Postpone the linearization to the numerical scheme and use nonlinear formulations directly.
  - Derive new nonlocal regularizing data constraints, including also GCA-based and photometric invariant color spaces.
  - Combine different data constraints.

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joan.duran@uib.es, toni.buades@uib.es

Dept. Mathematics and Computer Science  
University of Balearic Islands, Mallorca, Spain

March 1st, 2017



**Universitat**  
de les Illes Balears

